

A test for cosmic distance duality

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Testing the cosmic distance duality relation (CDDR) constitutes an important task for cosmology and fundamental physics since any violation of it would be a clear evidence of new physics. In this paper, we propose a new test for the CDDR using only current measurements of the gas mass fraction of galaxy clusters from Sunyaev-Zeldovich (f_{SZE}) and X-ray surface brightness (f_{X-ray}) observations. We show that the relation between f_{X-ray} and f_{SZE} observations is given by $f_{SZE} = \eta f_{X-ray}$, where η quantifies deviations from the CDDR. Since this latter expression is valid for the same object in a given galaxy cluster sample, the method proposed removes possible contaminations from different systematics error sources and redshift differences involved in luminosity and angular diameter distance measurements. We apply this cosmological model-independent methodology to the most recent f_{X-ray} and f_{SZE} data and show that no significant violation of the CDDR is found.

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I. INTRODUCTION

The so-called cosmic distance duality relation (CDDR) [1], which is closely connected with the Etherington reciprocity theorem [2, 3], plays an important role in observational cosmology ranging from gravitational lensing studies, galaxy and galaxy clusters observations, to analyses of the cosmic microwave blackbody radiation (CMB)[4]. It relates the luminosity distance D_L with the angular diameter distance D_A through the identity

$$\eta = \frac{D_L}{D_A}(1+z)^{-2} = 1. \quad (1)$$

The above result can be easily demonstrated for usual Friedmann-Lemaître-Robertson-Walker cosmologies. As is well known, it only requires source and observer to be connected by null geodesics in a Riemannian space-time and cosmological conservation of the number of photons [1, 3]. Examples of non-standard scenarios that violate the equality (1) are models in which photons do not travel on unique null geodesics, models with variations of fundamental constants or with photon non-conservation due to coupling to particles beyond the standard model of particle physics, absorption by dust, etc. (see, e.g., [9–11] for a discussion).

Attempts to actually test the CDDR validity from astronomical observations have started only recently [5, 7, 8]. For instance, in Ref. [5] it was argued that the Sunyaev-Zel'dovich effect (SZE) plus X-ray technique for measuring galaxy cluster distances is strongly dependent on the CDDR validity. As is well known, one can consider different electronic density dependencies combined with some assumptions about the galaxy cluster morphology

in order to evaluate its angular diameter distance with basis on Eq. (1), i.e. [6]

$$D_A^{data}(z) \propto \frac{(\Delta T_0)^2 \Lambda_{eH0}}{(1+z)^4 S_{X0} T_{e0}^2} \frac{1}{\theta_c}, \quad (2)$$

where $D_A^{data}(z)$ is the angular diameter distance to a given galaxy cluster obtained directly from a jointly analysis of its SZE and X-ray surface brightness observations, S_{X0} is the central X-ray surface brightness, T_{e0} is the central temperature of the intra-cluster medium, Λ_{eH0} is the central X-ray cooling function of the intra-cluster medium, ΔT_0 is the central decrement temperature, and θ_c refers to a characteristic scale of the cluster along the line of sight (l.o.s.), whose exact meaning depends on the assumptions adopted to describe the galaxy cluster morphology.

However, if one assumes a more general expression for CDDR, such that,

$$\frac{D_L}{D_A}(1+z)^{-2} = \eta(z), \quad (3)$$

it is possible to show that (for details see [5, 12])

$$D_A^{data}(z) = D_A(z)\eta(z)^2. \quad (4)$$

Therefore, $D_A^{data}(z)$ reduces to the real angular diameter distance only when the DD relation is strictly valid ($\eta \equiv 1$). In order to quantify the η parameter, the authors in Ref.[5] fixed $D_A(z)$ by using the cosmic concordance model [16] while for $D_A^{data}(z)$ they considered the 18 galaxy clusters from the Ref.[17] for which a spherically symmetric cluster geometry has been assumed. By assuming η constant, their statistical analysis provided $\eta = 0.91_{-0.04}^{+0.04} (1\sigma)$, and is therefore only marginally consistent with the standard result.

On the other hand, the CDDR should be tested only from astronomical observations, i.e., finding cosmological sources whose intrinsic luminosities and intrinsic sizes are

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known. Thus, after measuring the source redshift, one can determine both D_L and D_A to test directly the relation. In principle, this ideal method should not use any relationship coming from a specific cosmological model, i.e., they must be determined by means of intrinsic astrophysical quantities only.

In recent papers, the validity of the CDDR has been discussed using D_A measurements from galaxy clusters (GC) and luminosity distances from type Ia supernovae (SNe Ia) [8]. In such analyses, subsamples of GC and SNe Ia are built so that differences in redshift between objects in each sample are small ($\Delta z \simeq 10^{-3}$), thereby allowing a validity test of the CDDR. There are, however, at least three aspects that should be considered when performing this kind of analysis. First, that estimates of η from this method are potentially contaminated by different systematics error sources in GC and SNe Ia observations. Second, that some SNe Ia light curve fitters use a specific cosmological scenario in their calibration process, which makes the GC/SNe Ia test not completely model-independent (see Ref. [12] for a discussion on influence of the SNe Ia light curve fitters on the CDDR test). Finally, that the use of different objects to derive D_L and D_A implies necessarily in a choice for Δz , which affects the resulting estimates of the η parameter (see [13] for a recent discussion. See also [14] for an interesting model-independent CDDR test based on future observations of a redshifted 21cm signal from disk galaxies).

In order to circumvent these observational problems, we propose a consistent model-independent test for Eq. (1) that uses only current observations of the gas mass fraction of GC's. To perform our analysis, we use a sample of f_{X-ray} and f_{SZE} measurements of 38 GC's as discussed in Ref. [15]. We show that when the CDDR is taken into account current f_{X-ray} and f_{SZE} measurements provide a direct test for the CDDR.

II. GAS MASS FRACTION

A. X-ray Observations

The gas mass fraction is defined as $f = M_{gas}/M_{Tot}$ [19], where M_{Tot} is the total mass obtained via hydrostatic equilibrium assumption and M_{gas} (gas mass) is obtained by integrating the gas density model. The gas density model frequently used is the 3-dimensional electron number density of the spherical β model given by [20]¹

$$n_e(\mathbf{r}) = n_{e0} \left(1 + \frac{r^2}{r_c^2} \right)^{-3\beta/2}, \quad (5)$$

where n_e is the electron number density, r is the radius from the center of the cluster, r_c is the core radius of the intracluster medium (ICM), and β is a power law index. Under this assumption, M_{tot} and M_{gas} are given by [21]

$$M_{total}(r) = \frac{3\beta k_B T_e}{G\mu m_p} \frac{r^3}{r_c^2 + r^2}, \quad (6)$$

where T_e is the temperature of the intra cluster medium, μ and m_p are, respectively, the total mean molecular weight and the proton mass, k_B the Boltzmann constant, and

$$M_{gas}(r) = A \int_0^{r/D_A} \left(1 + \frac{\theta^2}{\theta_c^2} \right)^{-3\beta/2} \theta^2 d\theta, \quad (7)$$

where $A = 4\pi\mu_e n_{e0} m_p D_A^3$, $\theta = r/D_A$ and μ_e is the mean molecular weight of the electrons.

The X-ray surface brightness over some frequency band is written as

$$S_x = \frac{D_A^2}{4\pi D_L^2} \int n_e n_H \Lambda_{eH} d\ell \quad (8)$$

where the integral is along the line of sight and Λ_{eH} is the X-ray cooling function, proportional to $T_e^{1/2}$ [22]. Note that the above equation depends explicitly on the ratio between the angular diameter and luminosity distances. Thus, if the CDDR is taken as valid [$\eta = 1$ in Eq. (1)] and a fiducial angular diameter distance, D_A^* , is assumed, the central electron density n_{e0} can be analytically obtained, i.e., [15]

$$n_{e0}^{X-ray} = \left(\frac{S_{x0} 4\pi(1+z)^4 \frac{\mu_H}{\mu_e} \Gamma(3\beta)}{\Lambda_{eH} D_A^* \pi^{1/2} \Gamma(3\beta - \frac{1}{2}) \theta_c} \right)^{1/2}, \quad (9)$$

providing the well-known relation in current X-ray gas mass fraction measurements $f_{X-ray} \propto D_A^{*3/2}$ [19] (in the above expression S_{x0} is the central surface brightness and $\Gamma(x)$ is the Gamma function).

However, if the validity of the CDDR is not previously assumed ($\eta \neq 1$), Eq. (9) is now rewritten as

$$n_{e0}^{X-ray} = \eta \left(\frac{S_{x0} 4\pi(1+z)^4 \frac{\mu_H}{\mu_e} \Gamma(3\beta)}{\Lambda_{eH} D_A \pi^{1/2} \Gamma(3\beta - \frac{1}{2}) \theta_c} \right)^{1/2}, \quad (10)$$

which clearly shows that gas mass fraction measurements extracted from X-ray data are affected by a possible violation of the CDDR scaling as

$$f_{X-ray}^{th} \propto \eta D_A^{3/2}. \quad (11)$$

B. Sunyaev-Zel'dovich Observations

The measured temperature decrement ΔT_{SZE} of the CMB due to the Sunyaev-Zel'dovich effect [23] is given by [15]

$$\frac{\Delta T_0}{T_{CMB}} = f(\nu, T_e) \frac{\sigma_T k_B}{m_e c^2} \int n_e T_e dl, \quad (12)$$

¹ It is worth mentioning that we have used f_{gas} data for the non-isothermal double β -model in our statistical analyses. The spherical β model is used in this section only for simplicity but without loss of generality for the test proposed.

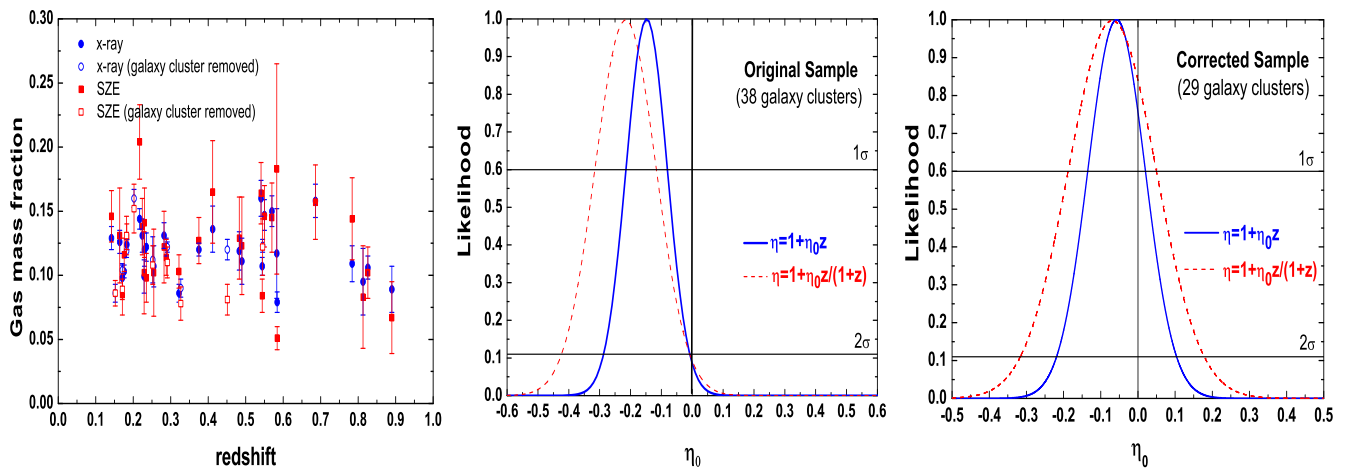


FIG. 1: **a)** Gas mass fraction measurements as a function of redshift for 38 galaxy clusters [15]. Blue circles (red squares) stand for measurements obtained via current X-ray (SZE) observations. Open circles and squares correspond to galaxy clusters that present large reduced χ^2 ($\chi^2_{d.o.f.} > 2$) when described by the hydrostatic equilibrium model. **b)** The likelihood distribution functions for the CDDR parameter η_0 . Blue solid lines correspond to the results for P1 whereas red dashed lines for P2. In this analysis, the entire sample of Ref. [15] was used (38 clusters). **c)** The same as in Panel 1b for the reduced galaxy cluster sample (29 clusters). When compared with the previous analysis, the compatibility of the data with the CDDR is clearly more evident.

where $T_{\text{CMB}} = 2.728$ K is the present-day temperature of the CMB, σ_T the Thompson cross section, m_e the electron mass and $f(\nu, T_e)$ accounts for frequency shift and relativistic corrections [24]. It worth mentioning that the gas temperature, T_e , is insensitive to the validity of the CDDR since it is obtained through the shape of the X-ray spectra (bremsstrahlung spectra) and not through X-ray luminosity.

Using SZE observations, the central electron density can now be expressed as

$$n_{e0}^{\text{SZE}} = \left(\frac{\Delta T_0 m_e c^2 \Gamma(\frac{3}{2}\beta)}{f(\nu, T_e) T_{\text{CMB}} \sigma_T k_B T_e D_A \pi^{1/2} \Gamma(\frac{3}{2}\beta - \frac{1}{2}) \theta_c} \right), \quad (13)$$

which is insensitive to the validity of the CDDR. Therefore, current gas mass fraction measurements via SZE depend only on the angular diameter distance as

$$f_{\text{SZE}} \propto D_A. \quad (14)$$

C. $f_{\text{SZE}}/f_{\text{X-ray}}$ relation

Current $f_{\text{X-ray}}$ measurements have been obtained by assuming the validity of the CDDR [35, 36]. If, however, this is not the case, the real gas mass fraction from X-ray observations (Eq.(11)) should be related with the current observations by $f_{\text{X-ray}}^{\text{th}} = \eta f_{\text{X-ray}}$. So, if all the physics behind the X-ray and SZE observations are properly taken into account, one would expect f_{gas} measurements from both techniques to agree with each other

since they are measuring the very same physical quantity. In this way, it is clear that the general expression relating current X-ray and SZE observations is given by:

$$f_{\text{SZE}} = f_{\text{X-ray}}^{\text{th}} = \eta f_{\text{X-ray}}, \quad (15)$$

which provides a direct test for the CDDR. Since the above expression holds for a given object, a possible influence on the η estimates due to redshift differences of distinct objects (e.g., in tests involving SNe Ia and GC) is fully removed.

Before discussing our estimates on the η parameter from the above relation, it is worth mentioning that in both methods to test the CDDR (the one of Eq. (15) and those using D_A measurements of galaxy clusters from their SZE and X-ray observations [5, 8, 12]) other physical effects could be present in the SZE observations, leading to results with $\eta \neq 1$ that not necessarily would be related to a CDDR violation in X-ray frequency. For instance, as mentioned earlier [see Eq. (13)], SZE observations are independent of the CDDR validity. However, these observations are redshift-independent only if there is no process of energy injection into the CMB. Otherwise, the standard linear relation of the CMB temperature evolution, $T_{\text{CMB}}(z) = T_{\text{CMB}}(z=0)(1+z)$, is not valid [25] and changes are needed in Eqs. (12) and (13). In this line, there are recent analyses testing the standard evolution of the CMB temperature through different techniques by using a more general expression such as $T_{\text{CMB}}(z) = T_{\text{CMB}}(z=0)(1+z)^{1+\beta}$ [26–30]. As a basic result, all analyses confirm the standard relation, i.e., $\beta \approx 0$ (see Ref. [30] for recent constraints on β value by using SZE observations and carbon monoxide excitation at high- z).

In what follows, we explore Eq. (15) and discuss constraints on the CDDR from current X-ray and SZE gas mass fraction measurements.

III. GALAXY CLUSTER DATA

To discuss the potential of Eq. (15) in probing the CDDR, we use the most recent X-ray and SZE gas mass fraction measurements to date, as given in Ref. [15]. The sample consists of 38 massive galaxy clusters spanning redshifts from 0.14 up to 0.89 whose X-ray data were obtained from the Chandra X-ray Observatory and SZE data from the BIMA/OVRO SZE imaging project, which uses the Berkeley-Illinois-Maryland Association (BIMA) and Owens Valley Radio Observatory (OVRO) interferometers to image the SZE. In order to perform a realistic model for the cluster gas distribution and take into account a possible presence of cooling flow, the gas density was modeled with the non-isothermal double β -model that generalizes the single β -model profile given by Eq. (5).

Another important aspect worth mentioning is that the shape parameters of the gas density model (θ_c and β) were obtained from a joint analysis of the X-ray and SZE data [31], which makes the SZE gas mass fraction not independent. However, as has been shown from current simulations [32], the values of θ_c and β computed separately by SZE and X-ray observations agrees at 1σ level within a radius r_{2500} (at which the mean enclosed mass density is equal to 2500 cosmological critical density), the same used in the La Roque *et al.* observations. Therefore, we believe that, unless a violation of the CDDR may affect even the shape parameters obtained from the X-ray data (and not only S_{x0}), one can use the La Roque *et al.* sample to perform the method discussed earlier. In Figure 1a we show the sample of 38 measurements of the gas mass fraction of galaxy clusters obtained via X-ray surface brightness (assuming $\eta = 1$) and the SZE (see also Table 5 of Ref. [15]).

IV. ANALYSIS AND RESULTS

We modify the CDDR to test for any violation by using simple parameterizations for η . In order to take into account a possible influence of different η parameterizations on the results, we use in our analyses two functions [18]:

$$\eta(z) = \begin{cases} 1 + \eta_0 z & \text{(P1)} \\ 1 + \eta_0 z / (1 + z) & \text{(P2)} \end{cases}$$

P1 is a continuous and smooth one-parameter linear expansion, whereas P2 includes a possible epoch-dependent correction, which avoids the divergence at very high z . Note also that, differently from constant η parameterizations (e.g., $D_L/D_A = \eta_0(1+z)^2$ [7]), both P1 and P2

recover the CDDR in the limit $z \rightarrow 0$, as obtained from cosmographic derivations.

We evaluate our statistical analysis by defining the likelihood distribution function $\mathcal{L} \propto e^{-\chi^2/2}$, where

$$\chi^2 = \sum_{i=1}^N \frac{[\eta(z) - \eta_{i,obs}(z)]^2}{\sigma_{i,obs}^2}, \quad (16)$$

$\eta_{i,obs}(z) = f_{SZE}/f_{X-ray}$ and $\sigma_{i,obs}^2$ is the uncertainty associated to this quantity. Also, the i index indicates the sum over each cluster. In Fig. 1b we plot our first constraints on the CDDR from Eq. (15). By considering all 38 galaxy clusters of the sample discussed earlier, we show the likelihood distribution as a function of the parameter η_0 for P1 (blue solid line) and P2 (red dashed line). For these two cases, we find $\eta_0 = -0.15 \pm 0.07$ ($\chi_{d.o.f.}^2 = 1.02$) and $\eta_0 = -0.22 \pm 0.10$ ($\chi_{d.o.f.}^2 = 1.04$) at 1σ level, respectively. Note that the CDDR ($\eta_0 = 0$) is slightly compatible with these data, being $\simeq 2\sigma$ off from the best-fit values in both cases. For the sake of completeness and also to verify the effect of the gas modelling on the results, we also performed the same analysis using the isothermal β model. In this case, we obtained $\eta_0 = -0.08 \pm 0.16$ and $\eta_0 = -0.13 \pm 0.21$ at 2σ level for the linear and non-linear parametrizations, respectively.

An important aspect concerning the galaxy cluster sample shown in Fig. 1a is that some objects present questionable reduced χ^2 ($2.43 \leq \chi_{d.o.f.}^2 \leq 41.62$) when described by the hydrostatic equilibrium model (see Table 6 in Ref. [34]). They are: Abell 665, ZW 3146, RX J1347.5-1145, MS 1358.4 + 6245, Abell 1835, MACS J1423+2404, Abell 1914, Abell 2163, Abell 2204. By excluding these objects from our sample (we end up with a subsample of 29 galaxy clusters), we perform a new analysis whose results are displayed in Fig. 1c. We note that, when compared with the previous analysis (Fig. 1b), the compatibility of the data with the CDDR is clearly more evident now, with $\eta_0 = -0.06 \pm 0.07$ (P1) and $\eta_0 = -0.07 \pm 0.12$ (P2) at 1σ level. It is worth observing that for all analyses performed in this paper a negative value for the CDDR parameter was preferred by the data (although the data are fully consistent with $\eta_0 = 0$). A possible explanation for that has been discussed in Ref. [10] in terms of cosmic opacity or the existence of axion-like and mini-charged particles (see, e.g., [11] for a recent review on these weakly-interacting-sub-eV particles). In Table I we summarize the main results of our analyses.

V. CONCLUSIONS

In this *Letter*, we have discussed how current measurements of gas mass fraction of galaxy clusters from X-ray and SZE effect observations can be used to test the CDDR. We have shown that if this relation is consistently taken into account current f_{X-ray} and f_{SZE} mea-

38 galaxy clusters data		$\chi^2/d.o.f$
P1	$\eta_0 = -0.15 \pm 0.14$	1.02
P2	$\eta_0 = -0.22 \pm 0.21$	1.04
29 galaxy clusters data		$\chi^2/d.o.f$
P1	$\eta_0 = -0.06 \pm 0.16$	0.93
P2	$\eta_0 = -0.07 \pm 0.24$	0.92

TABLE I: Constraints on the CDDR parameter η_0 . The error bars correspond to 2σ ($\Delta\chi^2 = 4$).

measurements are related by $f_{SZE} = \eta f_{X-ray}$ [Eq. (15)], allowing a direct test for the CDDR.

To perform our analyses we have considered two distinct forms for $\eta(z)$, i.e., $\eta = 1 + \eta_0 z$ and $\eta = 1 + \eta_0 z / (1 + z)$, which recover the equality between D_L and D_A at very low redshifts. By considering 38 gas mass fraction measurements of galaxy clusters obtained from their SZE and X-ray emissions, we have found no significant influence of the above $\eta(z)$ parameterizations on the results and that a no violation of the CDDR is compatible at 2σ level. However, if our sample is corrected to account for possible statistical error sources due to galaxy cluster modeling, a value compatible with the validity of the

CDDR is found (see Table I).

Finally, it is worth mentioning that, irrespective of the fiducial model adopted in observations, f_{gas} measurements from both techniques ($X-ray$ and SZE) for a given cluster must agree and if they do not the detected difference must be associated with the duality parameter η . Therefore, the method here proposed is cosmological model-independent and also presents a clear advantage over tests involving different kinds of observations (e.g., GC and SNe Ia) since the above relation is analyzed for the same object in a given sample. We believe that when applied to upcoming observational data the method discussed here may be useful to probe a possible violation of the CDDR.

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